Electric Potential

- **U(r)** of test charge $\mathbf{q_0}$ in electric field generated by other source charges is proportional to q_0
- So **U(r)/$\mathbf{q_0}$** is **independent of $\mathbf{q_0}$** which allows us to introduce **electric potential (V)** **independent of $\mathbf{q_0}$**

$$\Delta V(\vec{r}) = \frac{\Delta U(\vec{r})}{q_0} \quad \Rightarrow \quad V(\vec{r}) = \frac{U(\vec{r})}{q_0}$$

taking the same reference point

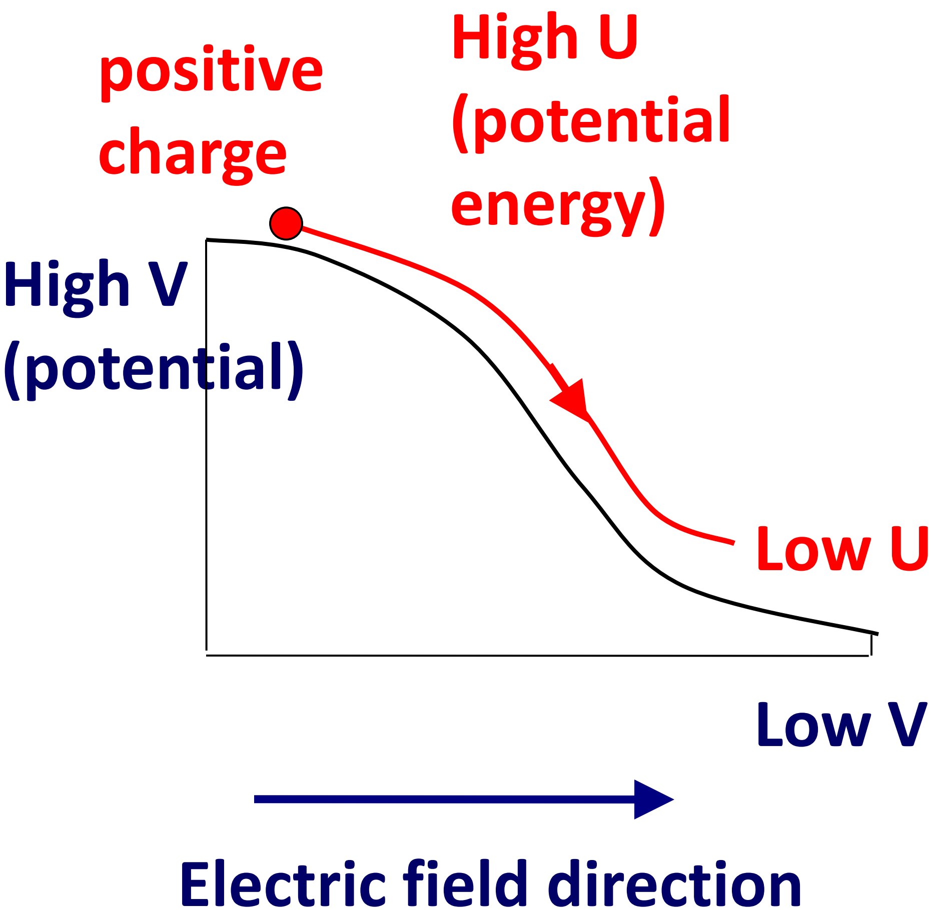
$$\Delta V = V_b - V_a = \frac{\Delta U}{q_0} = -\int_a^b \vec{E} \cdot d\vec{l}$$

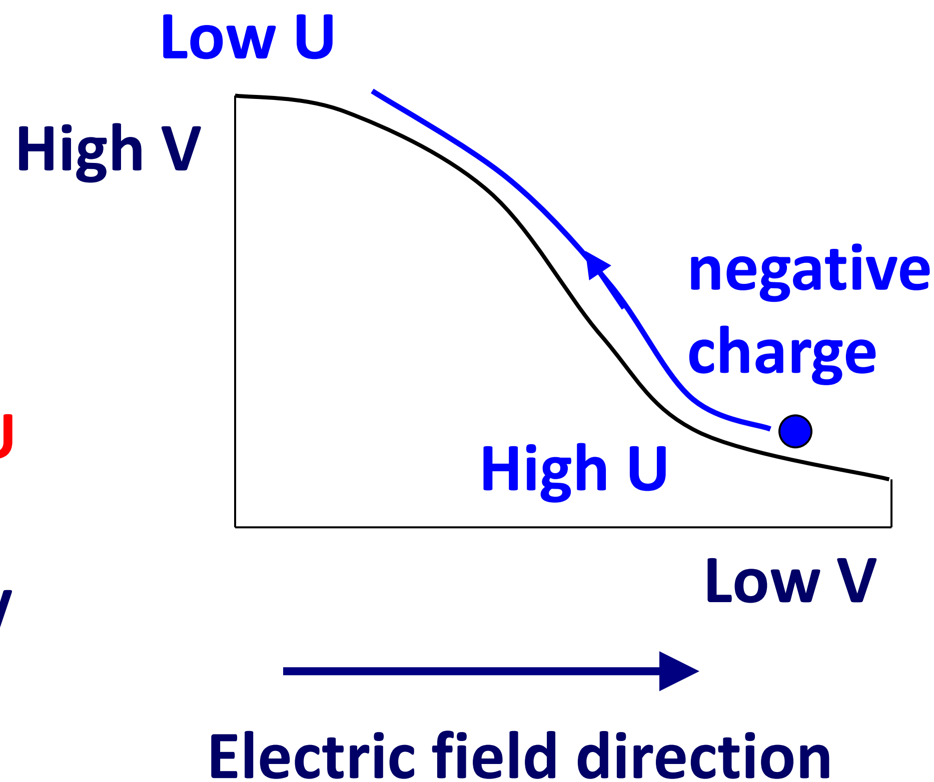
unit of electric potential is volt
1 V = 1 J/C

Electric Potential Energy and Electric Potential

$$V(r) = k\frac{q}{r}$$

- E points in the direction that V decreases most rapidly





Potential of Uniformly Infinite Plane of Charge

$$\Delta V = -\int_a^b \vec{E}\cdot d\vec{l}$$

- **What do we use as the reference point?**
  - **can't use infinity since plane is infinite**

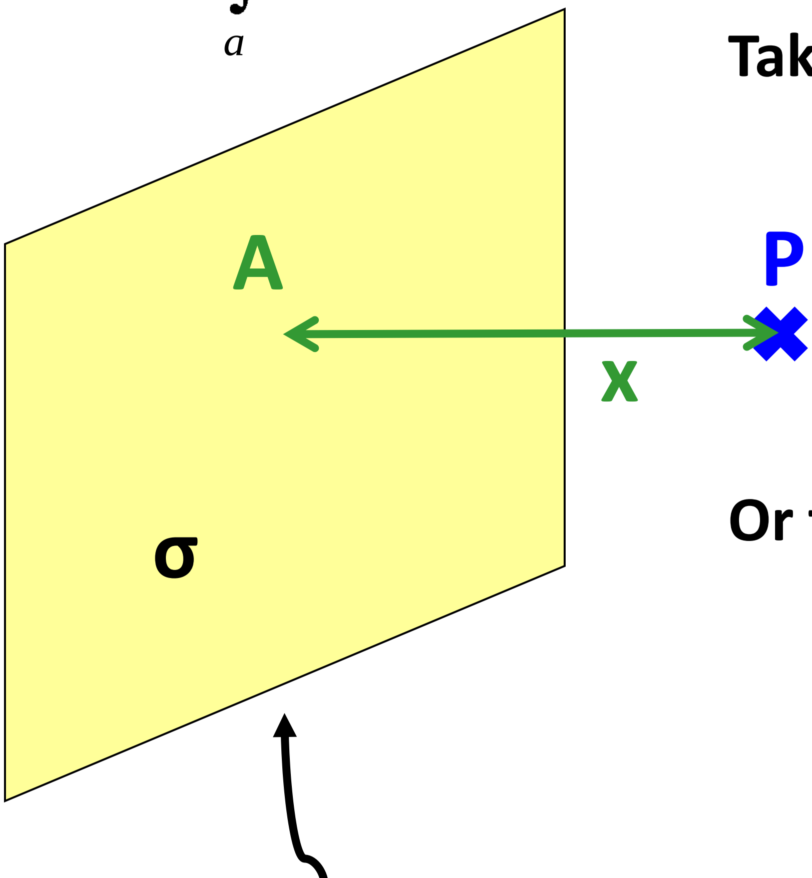
Take a reference point at A

$$\Delta V = -\int_A^P \vec{E}\cdot d\vec{l} = -\int_0^x \frac{\sigma}{2\varepsilon_0}\,dx$$

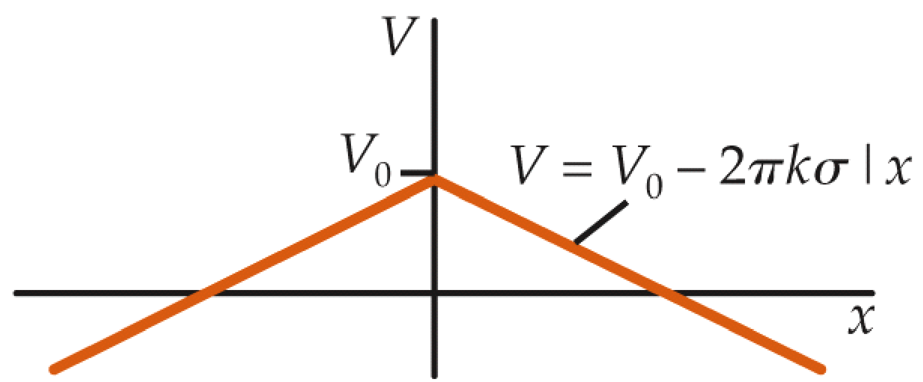
$$\Delta V = -\frac{\sigma}{2\varepsilon_0}\left|x\right| = -2\pi k\sigma\left|x\right|$$

Or take it at some other point so $V(0) = V_0$

$$\Delta V = -2\pi k\sigma\left|x\right| + V_0$$

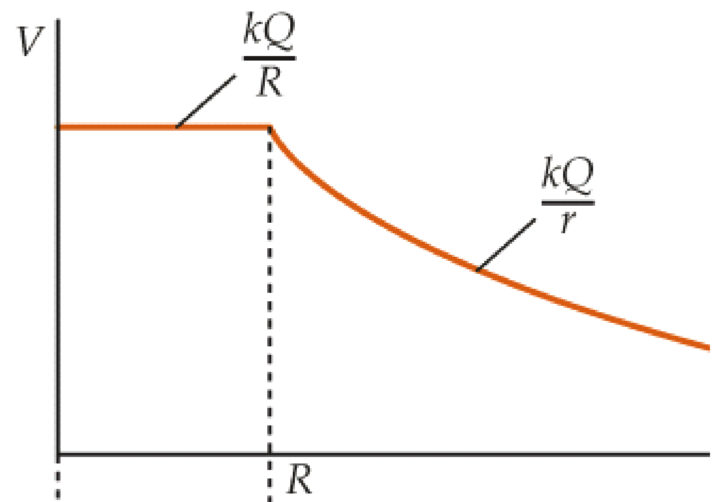


equipotential surface since V is constant



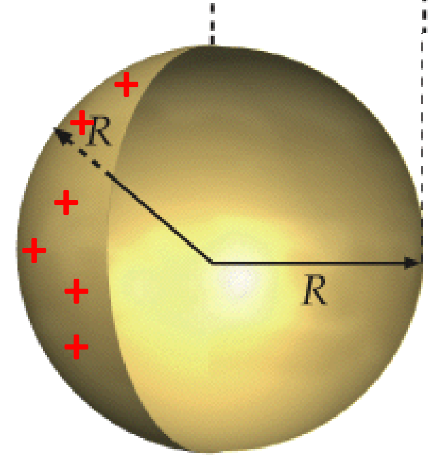
Potential of Uniformly Charged Spherical Shell

- **Electric Field (using Gauss's Law)**
  - **For r < R (inside),** $E = 0$
  - **For r > R (outside),** $E = k\dfrac{Q}{r^2}$



- **Potential**
  - **For r > R (outside),**

$$\Delta V = -\int_{\infty}^{r} \vec{E}\cdot d\vec{l} = -\int_{\infty}^{r} k\frac{Q}{r^2}\,dr = k\frac{Q}{r}$$

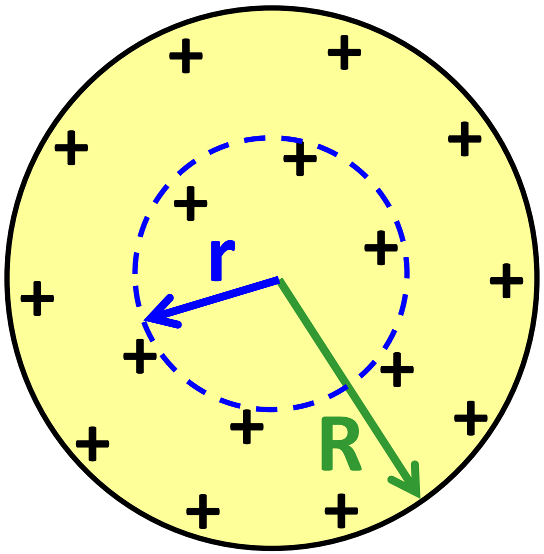


  - **For r < R (inside),**

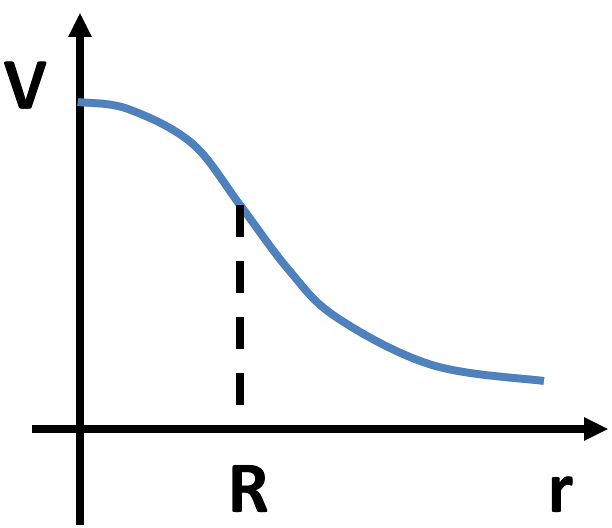
$$\Delta V = -\int_{\infty}^{r} \vec{E}\cdot d\vec{l} = -\int_{\infty}^{R} E\,dr - \cancel{\int_{R}^{r} E\,dr} = -\int_{\infty}^{R} k\frac{Q}{r^2}\,dr = k\frac{Q}{R}$$

same results for charged solid spherical conductor!

Potential of a Uniformly Charged Solid Sphere



insulator



- **For r > R (outside)**

$$\Delta V = -\int_{\infty}^{r} \vec{E} \cdot d\vec{l} = k\,\frac{Q}{r}$$

same result as shell or conducting sphere

$$E_{in} = k\,\frac{Q\,r}{R^3}$$

$$E_{out} = k\,\frac{Q}{r^2}$$

- **For r < R (inside)**

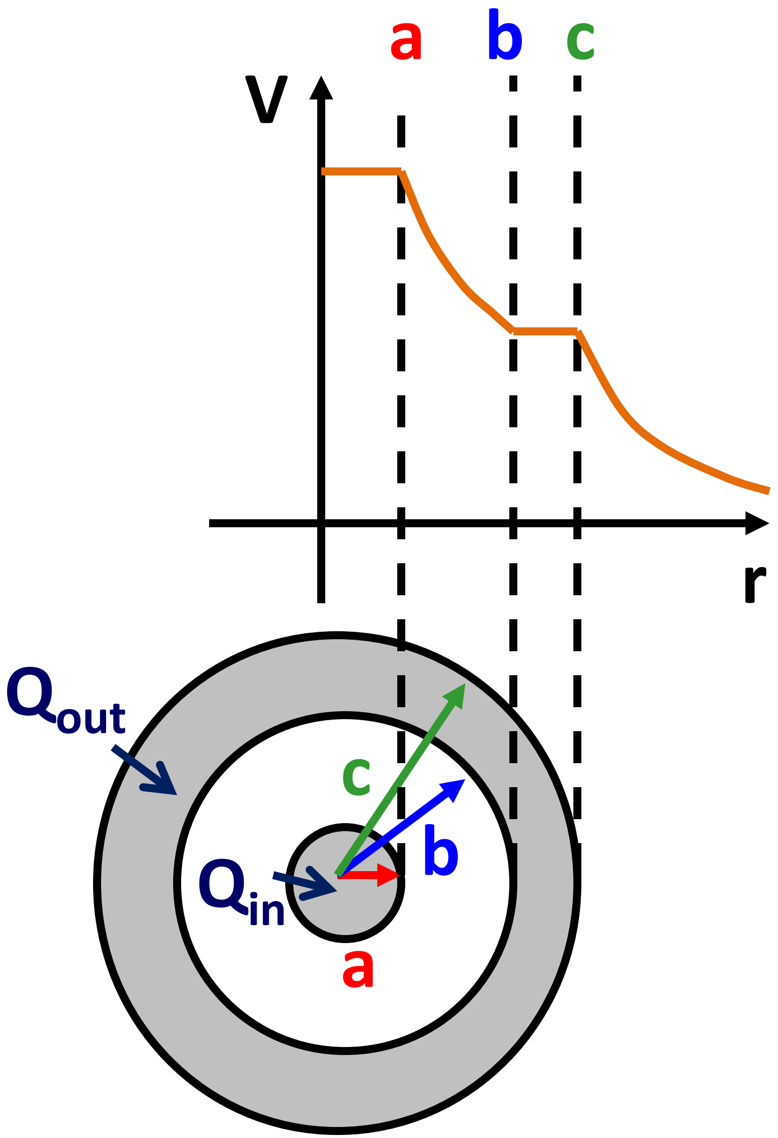
$$\Delta V = -\int_{\infty}^{r} \vec{E} \cdot d\vec{l} = -\int_{\infty}^{R} E_{out}\,dr - \int_{R}^{r} E_{in}\,dr$$

$$\Delta V = \frac{k\,Q}{R} - \int_{R}^{r} \frac{k\,Q}{R^3}\,r\,dr = \frac{k\,Q}{R} - \frac{k\,Q}{R^3}\left[\frac{r^2}{2}\right]_R^r$$

$$\Rightarrow \Delta V = \frac{k\,Q}{2R}\left(3 - \frac{r^2}{R^2}\right)$$

very different result as shell

Charged Concentric Spherical Conductors



- **For r > c**

$$V(r) = k\frac{Q_{in} + Q_{out}}{r}$$

- **For b < r < c**

$$V(r) = \text{constant}$$

- **For a < r < b**

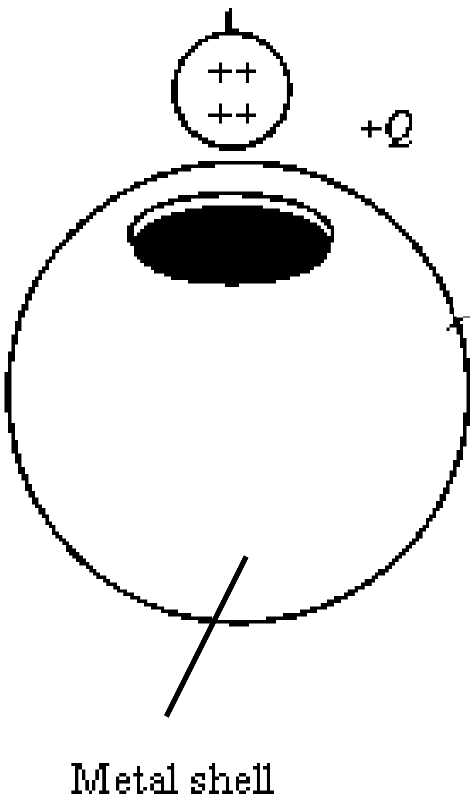
$$V(r) = k\frac{Q_{in}}{r} + \text{constant}$$

- **For r < a**

$$V(r) = \text{constant}$$

Quiz Question 1

A metal ball of charge +*Q* is lowered into an isolated, uncharged metal shell and allowed to rest on the bottom of the shell. When the charges reach equilibrium,



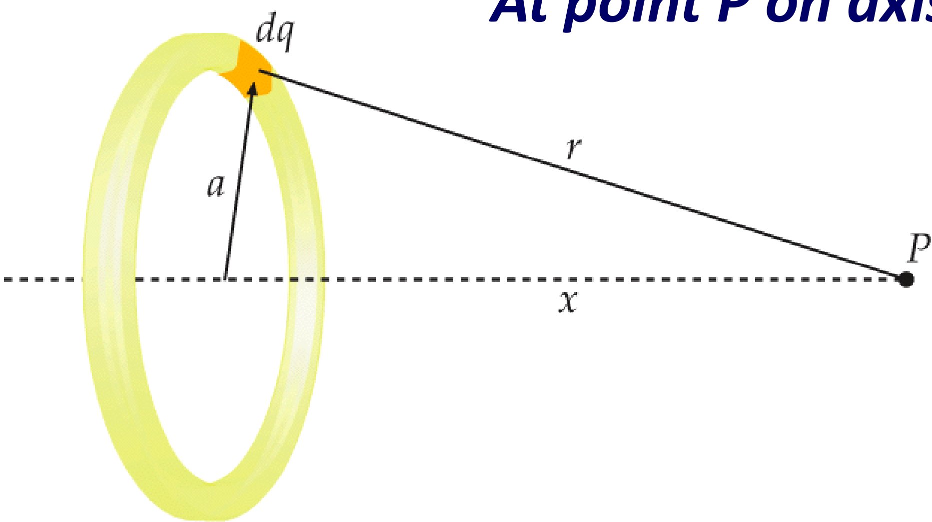
A. the outside of the shell has a charge of –*Q* and the ball has a charge of +*Q*.
B. the outside of the shell has a charge of +*Q* and the ball has a charge of +*Q*.
C. the outside of the shell has a charge of zero and the ball has a charge of +*Q*.
D. the outside of the shell has a charge of +*Q* and the ball has zero charge.
E. the outside of the shell has a charge of +*Q* and the ball has a charge of –*Q*.

Potential of Charged Ring

- **Potential from Continuous Charge Distribution:**

$$V = \int k \frac{dq}{r}$$

At point P on axis of ring

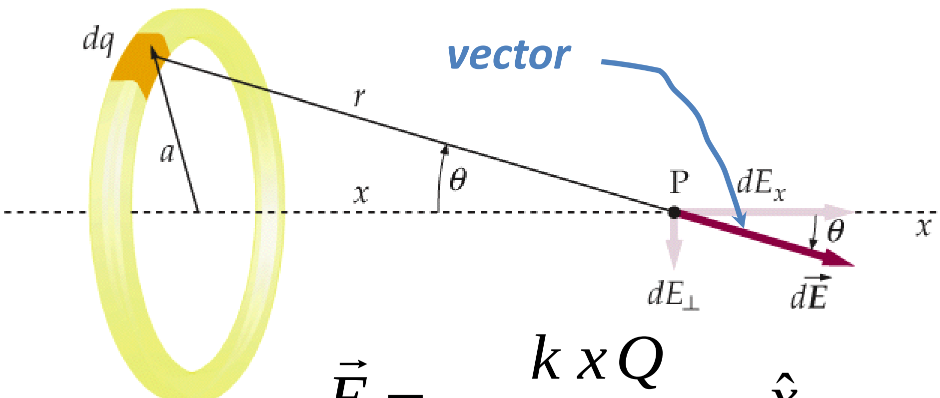


$$V = \int k \frac{dq}{r} = k \int \frac{\lambda\, a\, d\theta}{\sqrt{x^2 + a^2}}$$

$$V = k \frac{\lambda\, a}{\sqrt{x^2 + a^2}} \int_0^{2\pi} d\theta$$

$$V = k \frac{\lambda\, 2\pi\, a}{\sqrt{x^2 + a^2}}$$

$$\Rightarrow V = k \frac{Q}{\sqrt{x^2 + a^2}}$$

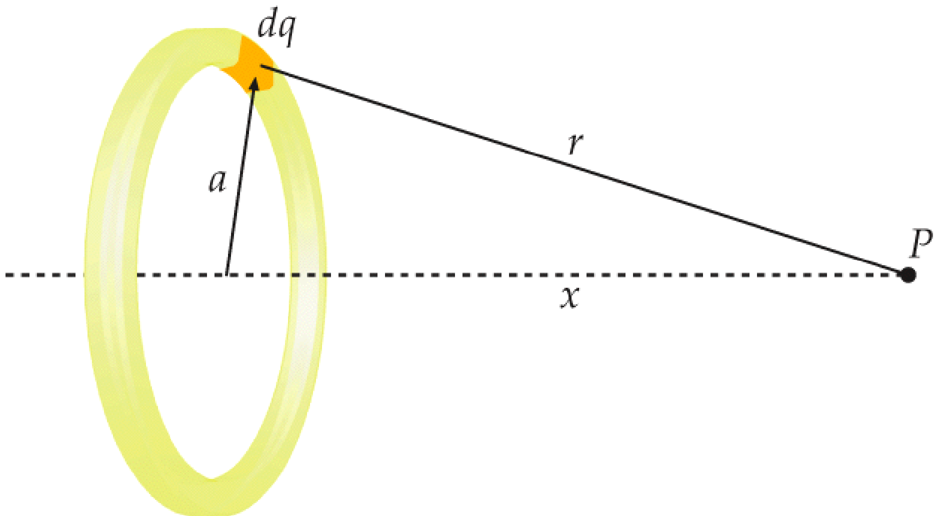


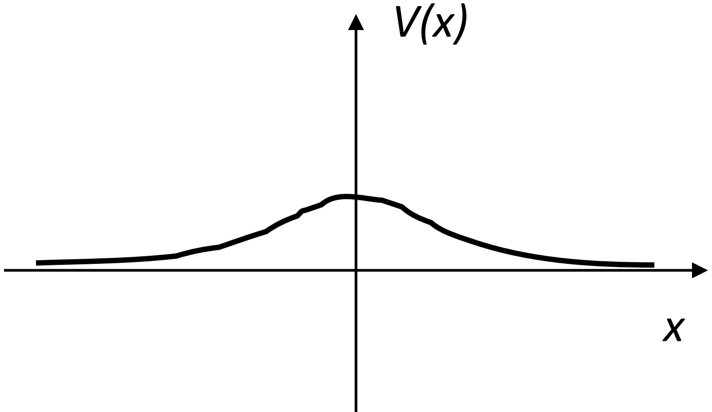
$$\vec{E} = \frac{k\, x\, Q}{(x^2 + a^2)^{3/2}}\, \hat{x}$$

Potential and Electric Field of Charged Ring

At point P on axis of ring

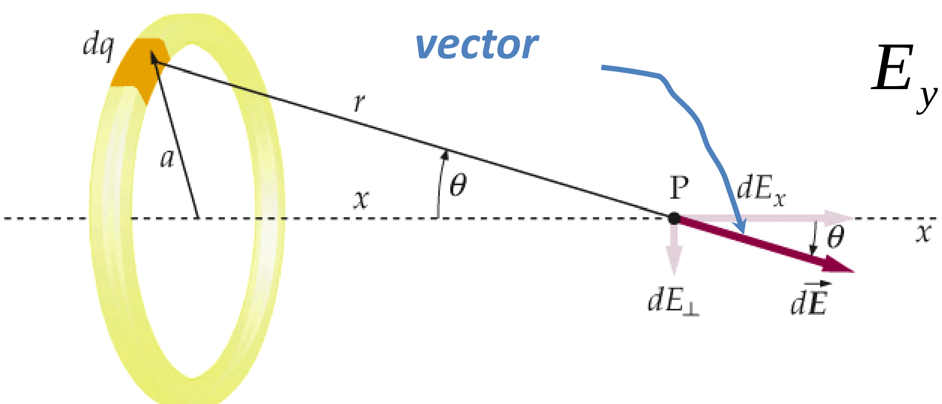
$$V = \frac{kQ}{\sqrt{x^2+a^2}} = \frac{kQ}{|x|}\frac{1}{\sqrt{1+(a/x)^2}}$$

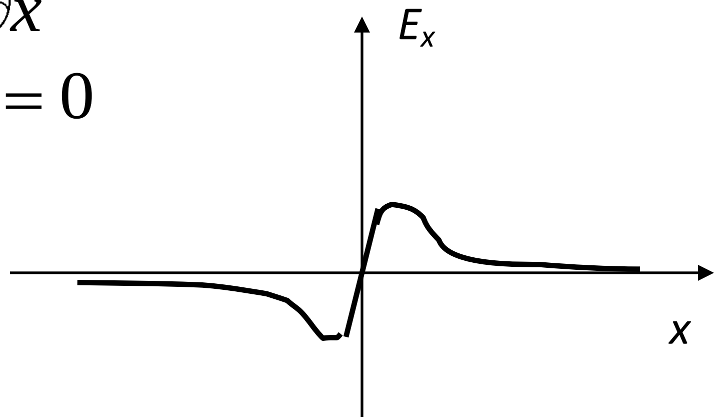




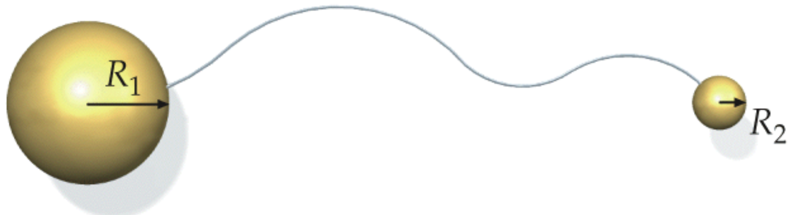
$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = E_z = 0$$





High Electric Field at Sharp Tips

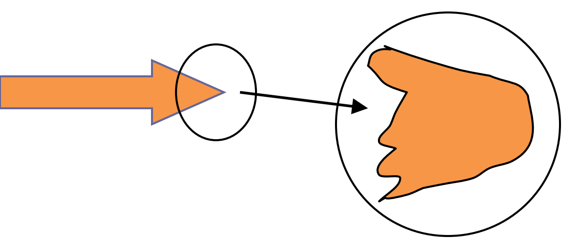


- Two **conducting** spheres are connected by a long **conducting** wire
- Total **charge** is $Q = Q_1+Q_2$

Potential is the same!

$$V_1 = \frac{kQ_1}{R_1} = \frac{kQ_2}{R_2} = V_2 \quad \Rightarrow \frac{Q_1}{Q_2} = \frac{R_1}{R_2}$$

With same potential, sphere with smaller radius has smaller amount of charge



$$V = \frac{Q}{4\pi\varepsilon_0 R} = \frac{4\pi R^2 \sigma}{4\pi\varepsilon_0 R} = \frac{R\sigma}{\varepsilon_0} \quad \Rightarrow \sigma = \frac{\varepsilon_0 V}{R}$$

$$E_1 = \frac{kQ_1}{R_1^{\ 2}}$$

$$E_2 = \frac{kQ_2}{R_2^{\ 2}} \quad \Rightarrow \frac{E_1}{E_2} = \frac{R_2}{R_1}$$

The smaller the radius of curvature, the larger the electric field

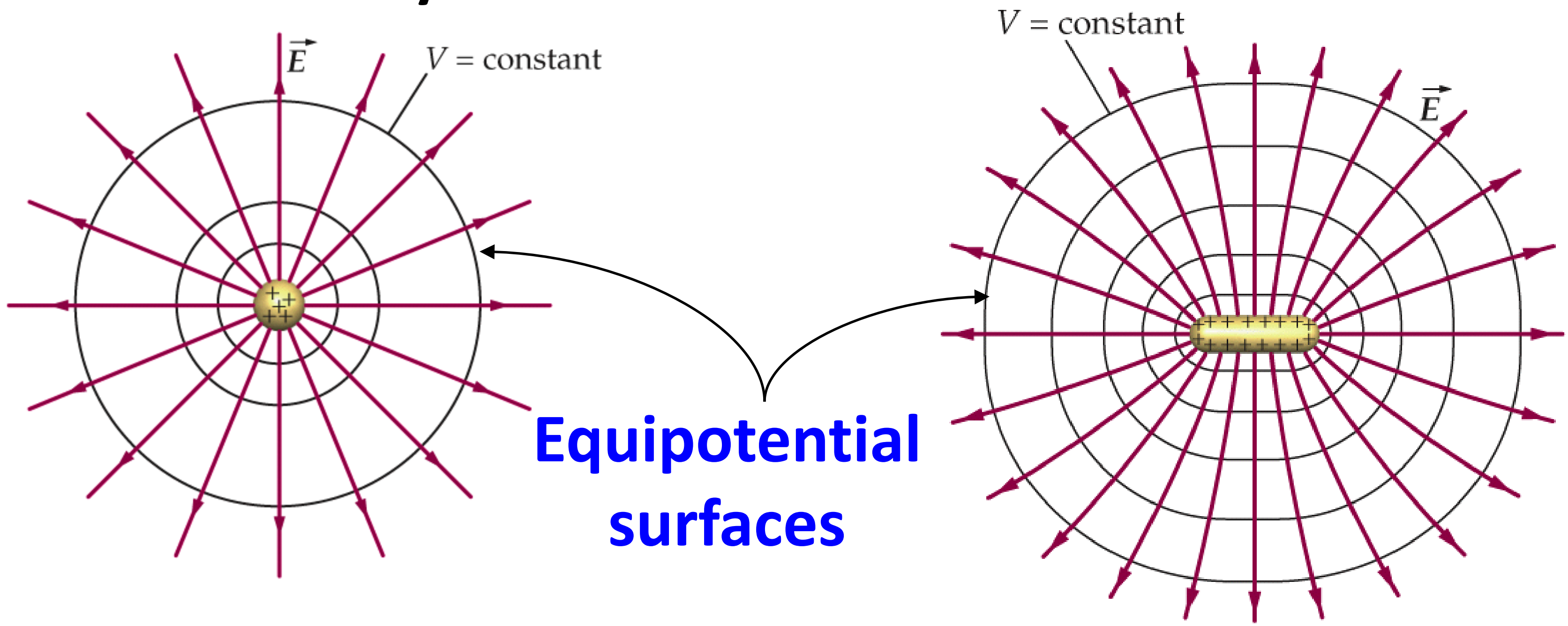
Quiz Question 2

An electric charge *q* is placed on an isolated metal sphere of radius r_1. If an uncharged sphere of radius r_2 (with $r_2 > r_1$) is then connected to the first sphere, the spheres will have equal

A. and like charges on their surfaces.
B. electric fields.
C. potentials.
D. capacitances.
E. but opposite charges on their surfaces.

Equipotential Surfaces

- An **equipotential surface** is a surface on which the potential is the same everywhere.

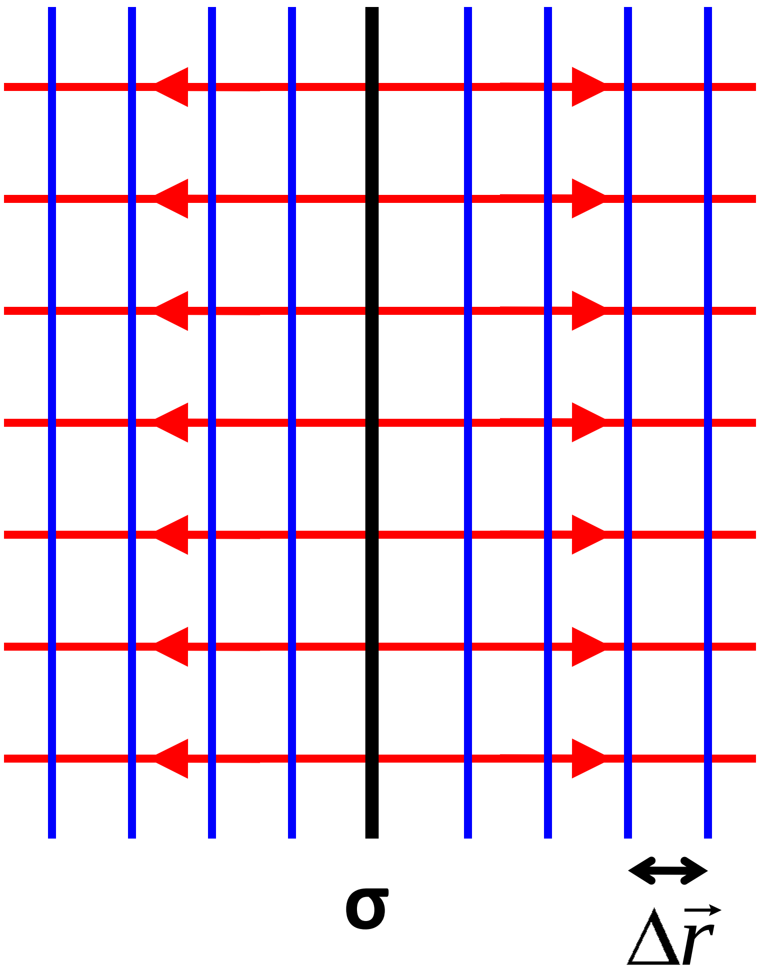


- **E** is perpendicular to **equipotential surfaces** everywhere
- **Equipotential surfaces** are drawn at constant intervals of **ΔV**
- **Potential difference** between nearby **equipotential surfaces** is approximately equal to **E** times the separation difference

$$\Rightarrow |\Delta V| = \left|\vec{E}\right| \left|\Delta \vec{r}\right|$$

Potential of a Uniformly Charged Sheet

$$E = \frac{\sigma}{2\varepsilon_0}$$



- **Electric field is uniform on each side of sheet as shown**
- **Equipotential surfaces are perpendicular to electric field**
- **Separation between equipotential surfaces are equal to the potential difference divided by the magnitude of electric field**

$$\Rightarrow |\Delta V| = |\vec{E}||\Delta\vec{r}|$$

Quiz Question 3

The vector that best represents the direction of the electric field intensity at point *x* on the 20 V equipotential line is

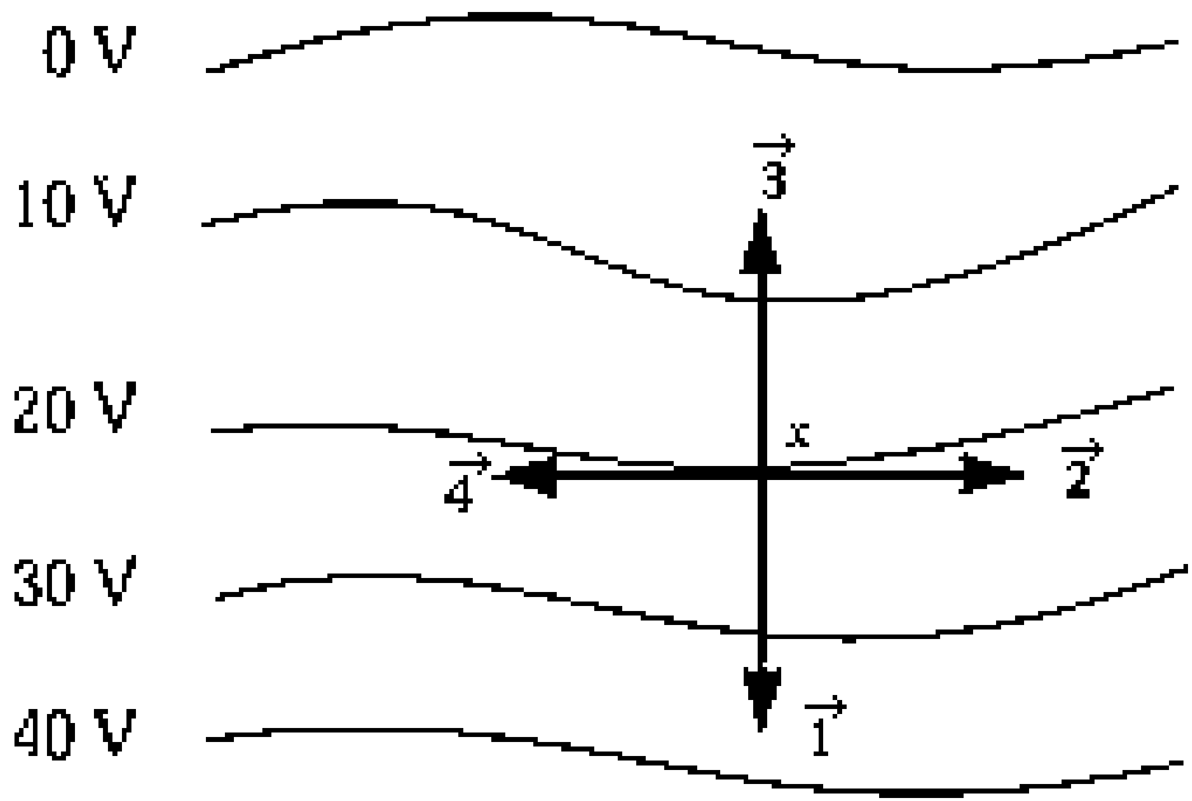
A. $\vec{1}$

B. $\vec{2}$

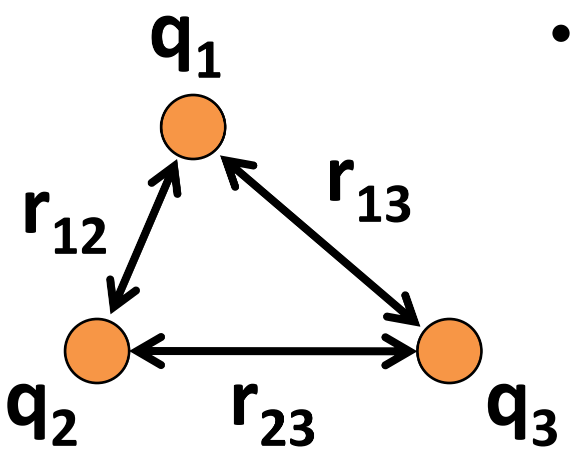
C. $\vec{3}$

D. $\vec{4}$

E. None of these is correct.



Electrostatic Potential Energy



- **The electrostatic potential energy of a system (relative to ∞) is the (external) work needed to bring the charges from an infinite separation to their final positions.**

$$U = W = \frac{k\,q_1\,q_2}{r_{12}} + \frac{k\,q_1\,q_3}{r_{13}} + \frac{k\,q_2\,q_3}{r_{23}}$$

$$U = \frac{1}{2}\left[q_1\left(\frac{k\,q_2}{r_{12}} + \frac{k\,q_3}{r_{13}}\right) + q_2\left(\frac{k\,q_3}{r_{23}} + \frac{k\,q_1}{r_{12}}\right) + q_3\left(\frac{k\,q_1}{r_{13}} + \frac{k\,q_2}{r_{23}}\right)\right]$$

$$U = \frac{1}{2}(q_1\,V_1 + q_2\,V_2 + q_3\,V_3)$$

In general for n charges:

$$U = \frac{1}{2}\sum_{i=1}^{n} q_i\,V_i$$

Quiz Question 4

The work required to bring a positively charged body from very far away is greatest for which point?